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REVIEW

# Improvement of water distribution networks analysis by topological similarity

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Principle of Quasi Work

**Abstract** In this research paper a methodology based on topological similarity is used to obtain starting point of iteration for solving reservoir and pipe network problems. As of now initial starting point for iteration is based on pure guess work which may be supported by experience. Topological similarity concept comes from the Principle of Quasi Work (PQW). In PQW the solution of any one problem of a class is used to solve other complex problems of the same class. This paves way for arriving at a unique concept of reference system the solution of which is used to obtain the starting point for starting the iteration process in reservoir and pipe network problems.


## Contents



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**Nomenclature**

| | |
|---|---|
| Bold letters | matrices |
| $D_i$ | diameter of the pipe '$i$'; $i$ = 1, 2, 3,... |
| EL | Elevation Level |
| $f_i$ | Darcy–Weisbach coefficient for pipe '$i$'. |
| $g$ | acceleration due to gravity |
| $\{h\}_n$ | head loss of pipe in a $TSS_n$; $n$ = 1, 2, 3, ... |
| $\{H\}_m$ | Head of reservoir in a $TSS_m$; $m$ = 1, 2, 3, ... |
| $h_i$ | head loss of pipe |
| $H_i$ | head of reservoir/ node; ($i$ = A, B, C). |
| $L_i$ | length of pipe line '$i$'. |
| $m, n$ | subscripts which represent different topological similar systems |
| $Q_i$ | discharge of pipe '$i$' |
| $\{Q\}_n$ | discharge of pipes in a $TSS_n$ |
| $r_i$ | resistance parameter for the pipe '$i$'. |

## 1. Introduction

Development of residential or industrial areas or discovery of new sources of supply leads to changes in the pattern of large scale pipe networks. This requires development of new robust analysis tools. Various methods for analysis of pipe networks available in the literature are Hardy-Cross method [1–4,18,19], Newton–Raphson [4,5] and linearization method [6,7], Ayad et al. [8] and Ates [9]. All the available methods solve a set of non-linear simultaneous equations iteratively beginning with an initial trial solution based on guess work. Guessing the initial solution for iterations is not easy especially for a beginner.

In this paper, for the first time concept of topological similarity as exclusively used in [10–15] to solve truss, beam and column problems with advantage is carried over into the domain of fluid mechanics (viz. reservoir and pipe networks). Pandita et al. [10] derived theorems, based on Principle of Quasi Work, useful for discrete structural models forming the lower end of finite elements. Topological similarity was advantageously used by Pandita et al. in [11] for obtaining redundant reactions of beam. A quick and simplified method for obtaining nodal deflections of an indeterminate truss using topological similarity is presented by Pandita [12]. An easy methodology for obtaining deflections of structures without resorting to internal forces/moments was given by Pandita and MarufWani [13]. Topological similarity was advantageously used by Pandita [14] to determine Euler critical load. Pandita [15] has derived the PQW and arrived at the definition of topologically similar systems. Topological similarity concept (through modified Bett's theorem) has also been successfully used to calculate the deflection of beam on an elastic foundation with advantage by Borak and Marcian [16]. As it has been successfully used in structural domain, here in this paper an attempt has been made to apply the same principle in the domain of fluid mechanics. Concept topological similarity was first demonstrated by Kheer [17] in the realm of fluid mechanics.

As PQW connects two topological similar systems the solution of one system can be used to arrive at the solution of all other system of the same class with ease. Hence, existence of a system whose solution can be used to solve all other topological similar problems exists. Here, such a system is designated as 'Reference system'. In this paper an attempt has been made to define such a reference system. Solution of this reference system is used for obtaining initial starting point for carrying out iteration in the cases of other reservoir and pipe network problems.

It is usually difficult to assess the initial trial values for head or for discharges. For example, in the case of three reservoir problem (Fig. 1), initial guess for the junction head '$H_J$' is usually made by taking average of highest and lowest heads of the reservoirs. If this guideline is applied to this problem then average of maximum and minimum comes out to be 170 m. This is equal to the head of second reservoir and the flow in pipe '2' is equal to zero. With this value of initial guess it will be difficult to arrive at the solution of the problem as it will take much more number of iterations. Hence, the problem is to resolve weather the guess should be less than 170 m or greater than 170 m in order to arrive at the final solution in minimum number of iterations. By present method, initial trial value of head for starting iterations comes out to be 168 m which is on the correct side (i.e. on the side of actual head of 148.8 m).

By present approach, guess work is completely eliminated and initial trial head/discharges in pipe network problems. This initial trial using topological similarity comes out to be quite closer to the actual solution and hence the numbers of iterations to arrive at the final solution are also considerably reduced.

## 2. Relations connecting two topologically similar networks

In this section general relation is derived for connecting two topological similar systems. These relations are used to calculate initial trial point in case of reservoirs and pipe network systems.

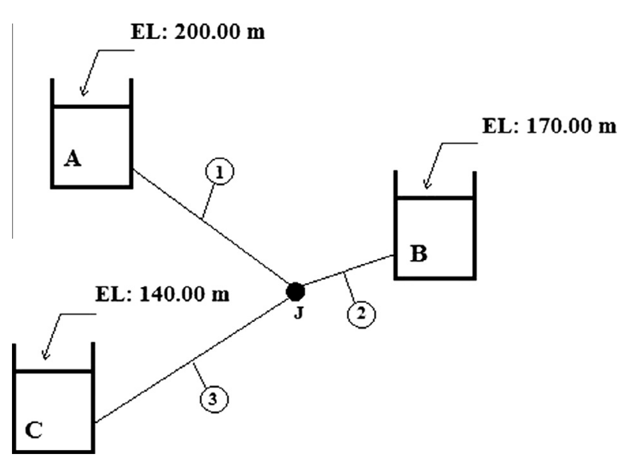


**Figure 1** Three reservoir problem.

### 2.1. Three and four reservoir system

Three and four reservoir systems generally solved in the literature are shown in Figs. 1 and 2. Pipes of these systems may have different value of parameters length '$L$', diameters '$D$' and Darcy–Weisbach friction factors '$f$'. The friction factors are usually taken constant. Reservoirs are connected by pipes through a common junction '$J$'. In these systems, reservoir '$A$' has the highest elevation and reservoir '$C$' the lowest; so that steady flow always takes place from reservoir $A$ to reservoir '$C$'. The direction of flow in pipe 2 in Fig. 1 and pipes 2 and 3 in Fig. 2 is not known and has to be calculated. The solution of these problems has to satisfy following two conditions:

1. Flow into each junction must be equal to flow out of the junction (Continuity equation) and
2. The Darcy–Weisbach equation

$$h = rQ^2 \tag{1}$$

$$\text{where } r = \frac{8fL}{\pi^2 g D^5} \tag{2}$$

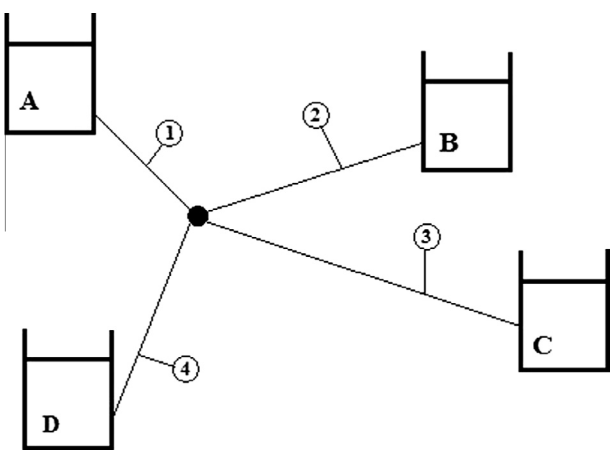


**Figure 2** Four reservoir problem.

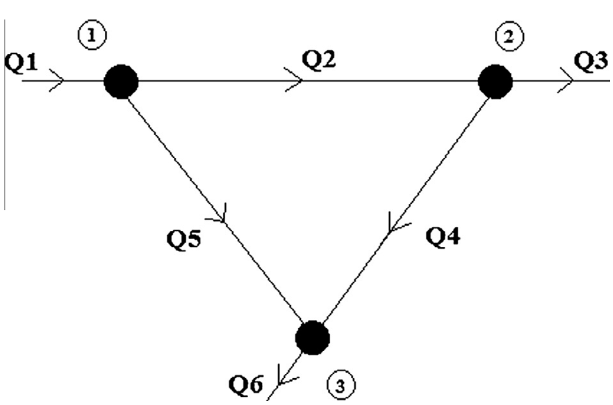


**Figure 3** Pipe network $(\mathrm{TSN})_1$.

### 2.2. Topologically similar network

Two systems are topologically similar, if these have same number of nodes and similar nodal interconnectivity. In this section an equation is derived, which connects two Topological Similar Networks (TSN).

Consider the network shown in Fig. 3 as a first network.

$$\text{At node 1} \quad -Q_1 + Q_2 + Q_5 = 0 \tag{3}$$

$$\text{At node 2} \quad -Q_2 + Q_3 + Q_4 = 0 \tag{4}$$

$$\text{At node 3} \quad -Q_4 - Q_5 + Q_6 = 0 \tag{5}$$

Now, Eq. (3) can also be written in the form of

$$\boldsymbol{A}\{Q\}_1 = 0 \tag{6}$$

$$\text{where } \boldsymbol{A} = \begin{bmatrix} -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix} \tag{7}$$

$$\text{And } \{Q\}_1^T = \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6,\} \tag{8}$$

Similarly,

$$h_1 = -H_1; \quad h_2 = H_1 - H_2; \quad h_3 = H_2;$$
$$h_4 = H_2 - H_3; \quad h_5 = H_1; \quad h_6 = H_3 \tag{9}$$

Or using Eq. (6)

$$\{\boldsymbol{h}\}_1 = \boldsymbol{A}^T\{\boldsymbol{H}\}_1 \tag{10}$$

Since all the relations from (3)–(10) are linear (involving only additions and subtractions), these will remain valid under any linear operation on $\{Q\}_1$, $\{H\}_1$ and $\{h\}_1$.

Now take the summation of branch multiple $h_k.Q_k$ for all $N$ branches of the network. Then, by Eqs. (10) and (6),

$$\sum_{K=1}^{N}\{h_k\}_1\{Q_k\}_1 = \{h_1\}^T\{Q\}_1 = \left[\boldsymbol{A}^T\{H\}_1\right]^T$$
$$= \left\{\boldsymbol{H}^T\right\}_1(\boldsymbol{A}\{Q\}_1) = 0 \tag{11}$$

Consider another network which has similar topological configuration $\mathrm{TSN}_2$ with same reference directions and numbering, the matrix $\boldsymbol{A}$ will remain same as for the network of Fig. 3. The parameters of second network will be denoted by $\{Q\}_2$, $\{H\}_2$ and $\{h\}_2$ then

$$\boldsymbol{A}\{Q\}_2 = 0 \tag{12}$$

$$\text{And } \{h\}_2 = \boldsymbol{A}^T\{H\}_2 \tag{13}$$

Next, let the physically meaningless quantity, $\sum_{k=1}^{N}\{h_k\}_1\{Q_k\}_2$ be found.

$$\sum_{K=1}^{N}\{h_k\}_1\{Q_k\}_2 = [\{h\}_1]^T\{Q\}_2 = \left[\boldsymbol{A}^T\boldsymbol{H}\right]^T\{Q\}_2$$
$$= [\{H\}_1]^T(\boldsymbol{A}\{Q\}_2) = 0 \tag{14}$$

where, Eqs. (10) and (12) were used.

$$\text{Similarly, } \sum_{k=1}^{N}\{h_k\}_2\{Q_k\}_1 = [\{h\}_2]^T\{Q\}_1 = 0 \tag{15}$$

As is now evident that Eqs. (14) and (15) written as $[\{h\}_1]^T\{Q\}_2 = [\{h\}_2]^T\{Q\}_1 = 0$ relates two topologically similar networks. Expanded form of Eqs. (14) and (15) is given as

$$\{h_A\}_m\{Q_A\}_n + \{h_B\}_m\{Q_B\}_n + \{h_C\}_m\{Q_C\}_n + \cdots + \{h_i\}_m\{Q_i\}_n = 0 \tag{16}$$

where $m, n$ = subscript 1, 2, 3, etc. represent different topological similar networks. Eq. (16) will be used to arrive at the initial trial values for head losses and consequently heads for reservoir problems.

However, in this research paper an empirical relation, Eq. (17), is also used to calculate the initial trial values of discharges in case of pipe network problems.

$$\frac{\{h_A\}_m}{\{Q_A\}_n} + \frac{\{h_B\}_m}{\{Q_B\}_n} + \frac{\{h_C\}_m}{\{Q_C\}_n} + \cdots + \frac{\{h_i\}_m}{\{Q_i\}_n} = 0 \tag{17}$$

### 2.3. Reference system for three reservoir problem

As has been already mentioned that the given problems will be solved using a solution of reference system. Here three reservoir system shown in Fig. 4 will be taken as reference system for solving other three reservoir problems and for getting initial flow discharges in pipe networks. It will be referred to as Topological Similar Network, $TSN_1$. The numerical values of discharge ($Q$) and head losses ($h$) of pipes and head ($H$) of each node are given in this figure.

For this $TSN_1$ Eq. (16) can be written as,

$$\{h_A\}_m\{Q_A\}_1 + \{h_B\}_m\{Q_B\}_1 + \{h_C\}_m\{Q_C\}_1 = 0 \tag{18}$$

where, subscript '$m$' represents some other TSN.

From Continuity $Q_A = Q_B + Q_C$ (19)

Substituting $Q_A = 0.13$, $Q_B = 0.04$ (as given in Fig. 4) we get $Q_C = 0.09$ from Eq. (19)

Now substituting values of $Q_A$, $Q_B$ and $Q_C$ in Eq. (18)

$$\{h_A\}_m 0.13 + \{h_B\}_m 0.04 + \{h_C\}_m 0.09 = 0 \tag{20}$$

And Eq. (17) takes the form of

$$\frac{\{h_A\}_m}{0.13} + \frac{\{h_B\}_m}{0.04} + \frac{\{h_C\}_m}{0.09} = 0 \tag{21}$$

Initial head at the junction for any three reservoirs problem will be calculated by using either Eq. (20) or Eq. (21). Final solution is then obtained by iterations as used in conventional methods.

## 3. Three reservoir system

In this section an example of three reservoir problem will be taken up. It will be solved by first calculating the trial value of piezometric head ($H_J$) at junction of pipes by using reference system and then conventional method of iterations as given in [1–4] will be used for arriving at the final solution.

### 3.1. Steps to solve the reservoir problem

The steps involved to solve the reservoir problems after finding the initial trial value of piezometric head ($H_J$) in the conventional method are as follows:

(i) For each $H_J$ calculate $Q_i$ in each pipeline with positive sign if it is towards the junction and negative sign if away from the junction. Find $Q = \sum Q_i$ and also find $\sum \frac{Q}{h}$.

(ii) The additive correction, to be added to the calculated value of $H_J$ for purposes of the next trial, is

$$\Delta H_J = \frac{2\Delta Q}{\sum \left[\frac{Q}{h}\right]} \tag{22}$$

(iii) New $H_J$ for next trial is calculated as

$$(H_J)_{n+1} = (H_J)_n + (\Delta H_J)_n \tag{23}$$

(iv) Continue till $H_J$ converges to the desired accuracy.

**Example 1** (*Three Reservoir System*). Here solution of a three reservoirs system shown in Fig. 5 will be obtained. This system

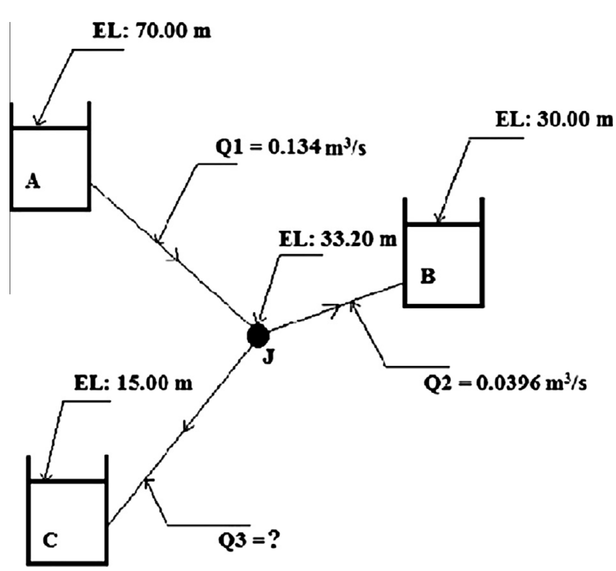


**Figure 4** Three reservoir system (reference system): $TSN_1$.

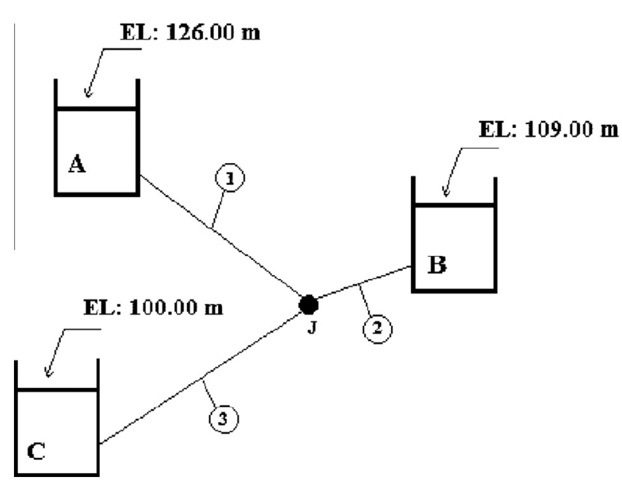


**Figure 5** Three reservoir system: $TSN_2$.

**Table 1** Details of Example 4.2.1.

| Pipe | Diameter (cm) | Length (m) | Connectivity |
|---|---|---|---|
| 1 | 15 | 350 | AJ |
| 2 | 10 | 200 | BJ |
| 3 | 10 | 250 | CJ |

will be referred as $TSN_2$. Table 1 shows the details of the system. Let its head at junction $J$ be $H_J$. The value of friction factor, '$f$', is taken as 0.02 for all the pipes.

The value of head at junction $J$ is calculated by using Eq. (20) of $TSN_1$

$$\{h_A\}_m 0.13 + \{h_B\}_m 0.04 + \{h_C\}_m 0.09 = 0 \tag{24}$$

$$[126 - \{H_J\}_2]0.13 + [109 - \{H_J\}_2]0.04 + [100 - \{H_J\}_2]0.09 = 0 \tag{25}$$

$$\{H_J\}_2 \cong 114 \tag{26}$$

This value of $H_J$ is taken as the starting point of iterations.

Now, the values of resistance parameters for pipe '1' using Eq. (2) are given by

$$r_1 = \frac{8fL_1}{\pi^2 g D_1^5} = \frac{8 \times 0.02 \times 350}{\pi^2 \times 9.81 \times (0.15)^5} = 7617 \tag{27}$$

Similarly $r_2$ and $r_3$ are calculated and their values are 33,051 and 41,313 respectively. Flow away from the junction is taken as negative.

*First trial:* Now for the first trial, the value of head junction ($H_J$) is 114.0 m as given in Eq. (26).

| Pipe | $r$ | Estimated $h_J$ (m) | $Q = \sqrt{\left(\frac{h_J}{r}\right)}$ m$^3$/s | Q (L/s) | $\left[\frac{Q}{h_f}\right]$ |
|---|---|---|---|---|---|
| 1 | 7617 | 126.0–114.0 = 12.0 | +0.0397 | + 39.7 | 3.31 |
| 2 | 33,051 | 114.0–109.0 = 5.0 | −0.0123 | −12.3 | 2.46 |
| 3 | 41,313 | 114.0–100.0 = 14.0 | −0.0184 | −18.4 | 1.31 |
| $\Delta Q = 9.0$ | | | | | $\sum\left[\frac{Q}{h_f}\right] = 7.08$ |
| Additive Correction = 2.54 | | | | | |

And the value of $H_J$ for next trial using Eq. (23) is given by

$$H_J = 114.0 + 2.54 = 116.54 \tag{28}$$

In a similar way the value of additive correction ($\Delta H_J$) of second trial comes out to be 0.02928, which is very small. Hence no further iteration is necessary. And the value of piezometric head at junction $J$ = 116.57 m. Error in the discharge after this iteration is 0.1 L/s (0.0001 m$^3$/s). The iterations are stopped and the finalized discharges are $Q_1 = 35.2$ L/s, $Q_2 = -15.10$ L/s (towards reservoir $B$), $Q_3 = -20.00$ L/s (towards reservoir $C$).

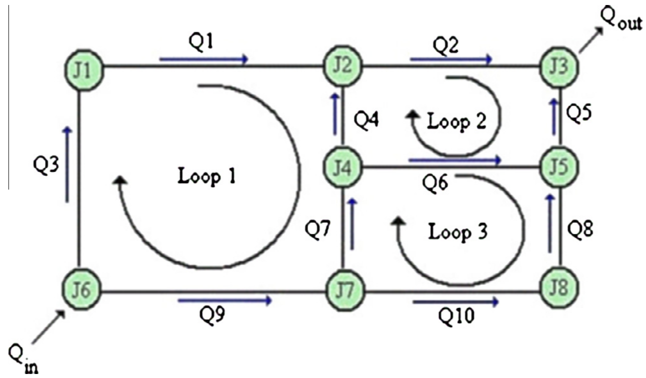


**Figure 6** Pipe network.

It can be seen that the initial trial head calculated by this approach is equal to the initial trial value (pure guess work) given in [4]. This is quite close to the final answer and only two trials were needed for reaching the answer.

## 4. Pipe networks

Interconnected pipe network through which the flow to a given outlet may come from several loops is called pipes networks as shown in Fig. 6. In this paper fluid flow direction of loops is taken positive in clockwise direction. These problems in general need experience for guessing initial discharge in pipes of the network. Each trial solutions should satisfy the two conditions of the flow given in Section 2.1.

### *4.1. Reference system for pipe network*

Since every pipe network problem has many nodes branches in which flow may be towards the node or away from the node. These nodes when isolated from the pipe networks are topological similar to the reservoir problems. Hence, the solution of any reservoir problem is used to obtain initial trial values of flow at the node. Here, three reservoirs of Fig. 4 are chosen as reference system $TSN_1$.

The equation for head loss and discharge at a given node having three pipe connections given by empirical Eq. (17) is as follows:

$$\frac{\{h_A\}_n}{\{Q_A\}_m} = \frac{\{h_B\}_n}{\{Q_B\}_m} + \frac{\{h_C\}_n}{\{Q_C\}_m} \tag{29}$$

From Fig. 4, the values of reservoir heads and junction head i.e. $H_A = 70$ m, $H_B = 30$ m, $H_C = 15$ m and $H_J = 33.20$ m are given. Using these values we obtained the head losses of reservoirs as given below:

$$\{h_A\}_1 = 36.8, \quad \{h_B\}_1 = 3.2 \quad \text{and} \quad \{h_C\}_1 = 18.2 \tag{30}$$

Then Eq. (30) becomes as follows:

$$\frac{36.8}{\{Q_A\}_m} = \frac{3.2}{\{Q_B\}_m(r\uparrow)} + \frac{18.2}{\{Q_C\}_m(r\downarrow)} \tag{31}$$

From Eq. (31), the initial trial value of discharge in each pipe is calculated as follows:

(i) Choose a node having 3 branches in which flow may be towards the node or away from the node (Continuity Equation). One discharge in any branch is already known in the problem.
(ii) Now using Eq. (31) and Continuity Equation determine the discharge in other two pipes.
(iii) The procedure is repeated for all nodes till we get the initial trial value for discharges in all the branches of the network.

### 4.2. Steps to solve the pipe network problem

After calculating the discharges in each pipe of network by using Eq. (31) of $TSN_1$, iterations can be carried over by using any conventional method. Here in this paper Hardy – Cross Method of Analysis as given in [1–4] is used. The steps involved in this method are as follows:

(i) Using the trial vales of discharges, head loss in each pipe is calculated as $h = rQ^n = rQ|Q^{n-1}|$ and also the quantity $(rn|Q^{n-1}|)$ is calculated for each pipe.

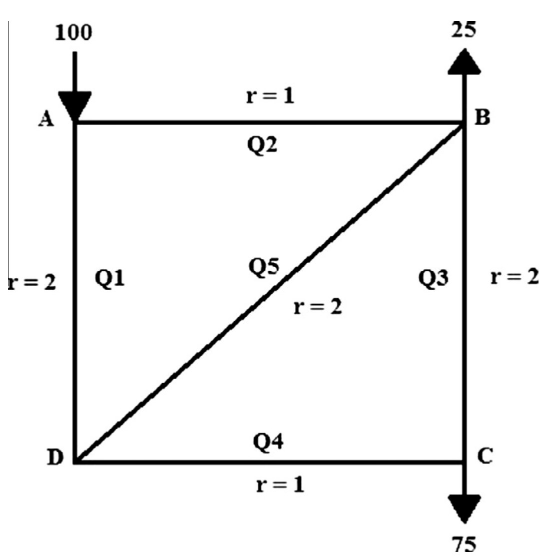


**Figure 7** Pipe network: $TSN_3$.

(ii) The quantity $\Delta Q = \frac{-\sum rQ^n}{\sum rn|Q^{n-1}|}$ is calculated for each loop. This represents the correction in discharges in the pipes of the loop.

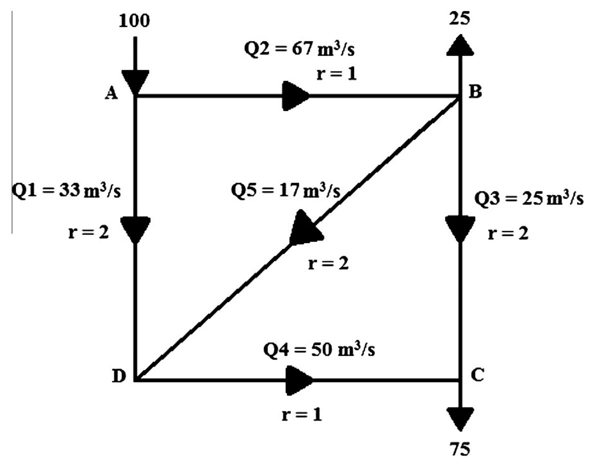


**Figure 7a** Pipe network: $TSN_3$.

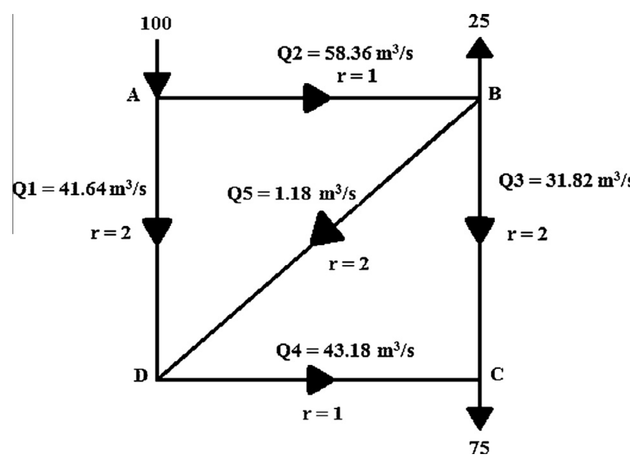


**Figure 7b** Pipe network: $TSN_3$.

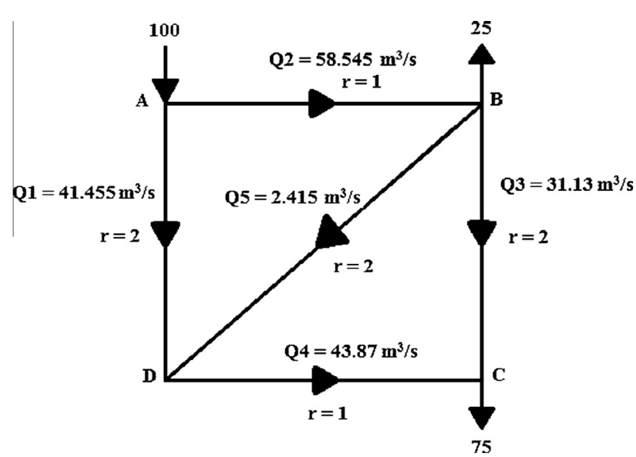


**Figure 7c** Pipe network: $TSN_3$.

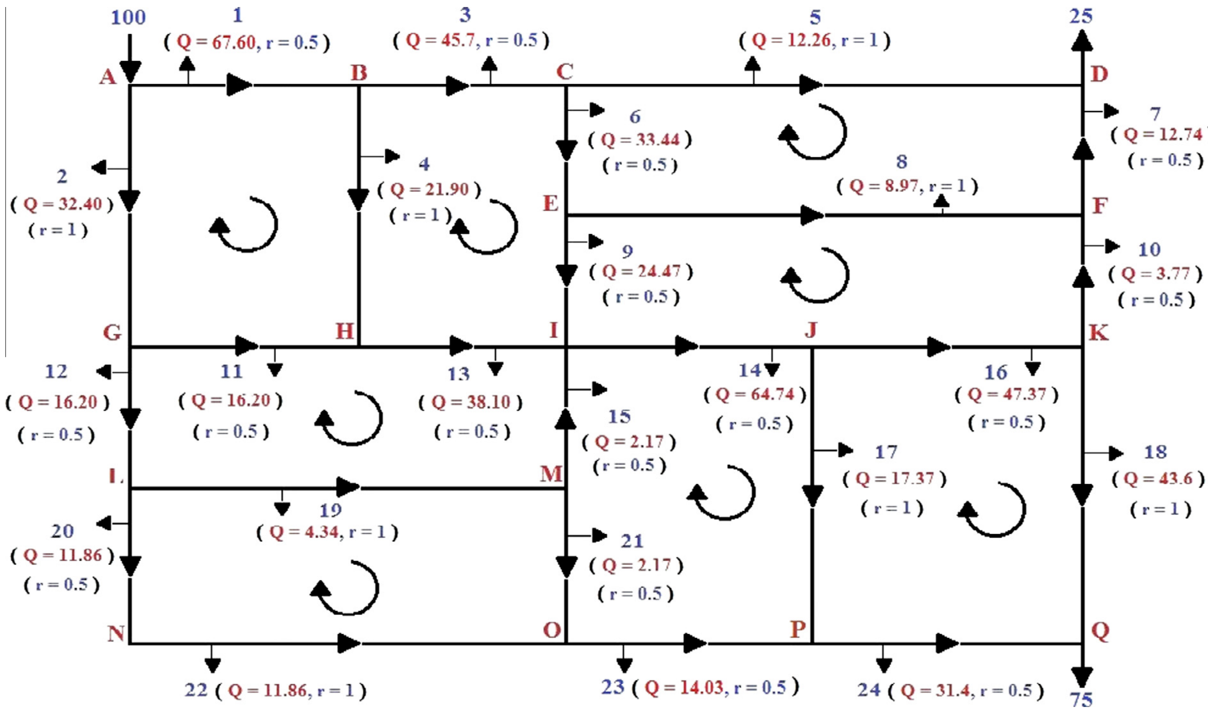


**Figure 8** Pipe network: $TSN_4$.

(iii) These corrections for the loops are applied to the pipes of the loops.
(iv) Steps (i) to (iii) are repeated till ΔQ is as small as desired.

### 4.3. Initial trial point for pipe network using reference system

In this section two examples for discharge through pipe networks are solved. Pipe networks having two and eight loops with different flowing conditions are solved starting with the initial trial point obtained by using reference system of three reservoirs Fig. 4.

**Example 2** (*Network with two loops*). A pipe network shown in Fig. 7, in which three flows are given is denoted as $TSN_3$. Let its discharge will be $Q_1$, $Q_2$, $Q_3$, $Q_4$ and $Q_5$ for pipes 1, 2, 3, 4 and 5 respectively as shown in the figure. The discharge in each pipe is calculated from reference Eq. (31) of $TSN_1$:

At junction A:

$$\frac{36.8}{\{Q_A\}_3} = \frac{3.2}{\{Q_B\}_3(r\uparrow)} + \frac{18.2}{\{Q_C\}_3(r\downarrow)} \tag{32}$$

$$\frac{36.8}{100} = \frac{3.2}{Q_1} + \frac{18.2}{Q_2} = \frac{3.2}{Q_1} + \frac{18.2}{100 - Q_1} \tag{33}$$

$$Q_1 = +26.83, +32.40 \tag{34}$$

$$\text{Let's take } Q_1 = 32.40 \approx 33 \tag{35}$$

$$\text{Then, } Q_2 = 100 - 33 = 67 \tag{36}$$

In a similar way the values of discharges at junction $C$ are obtained i.e.

$$Q_4 = +24.32 \approx 25 \tag{37}$$

$$\text{Then } Q_3 = 75 - 25 = 50 \tag{38}$$

By applying continuity Equation at node B we get $Q_5 = 17$.

Now the distribution of discharge is shown in Fig. 7a.

Flow direction is assumed positive clockwise for all loops. The Hardy- Cross method is used to find the corrections $\Delta Q$.

*First trial:* From Fig. 7a.

| LOOP ABD | | | LOOP BCD | | |
|---|---|---|---|---|---|
| LINE | $rQ^2$ | $\lvert 2rQ\rvert$ | LINE | $rQ^2$ | $\lvert 2rQ\rvert$ |
| AB | $1(67)^2 = 4489$ | 134 | BC | $2(25)^2 = 1250$ | 100 |
| BD | $2(17)^2 = 578$ | 68 | CD | $-1(50)^2 = -2500$ | 100 |
| DA | $-2(33)^2 = -2178$ | 132 | DB | $-2(17)^2 = -578$ | 68 |
| | $\sum rQ^2 = 2889$ | $\sum \lvert 2rQ\rvert = 334$ | | $\sum rQ^2 = -1828$ | $\sum \lvert 2rQ\rvert = 268$ |
| $\Delta Q = -8.64$ | | | $\Delta Q = 6.82$ | | |

**Table 2** Initial trial point of discharges in each pipe.

| Pipes | Discharge $Q$ ($m^3/s$) | Pipes | Discharge $Q$ ($m^3/s$) | Pipes | Discharge $Q$ ($m^3/s$) | Pipes | Discharge $Q$ ($m^3/s$) |
|---|---|---|---|---|---|---|---|
| 1 | 67.60 | 7 | 12.74 | 13 | 38.10 | 19 | 4.34 |
| 2 | 32.40 | 8 | 8.97 | 14 | 64.74 | 20 | 11.86 |
| 3 | 45.70 | 9 | 24.47 | 15 | 2.17 | 21 | 2.17 |
| 4 | 21.90 | 10 | 3.77 | 16 | 47.37 | 22 | 11.86 |
| 5 | 12.26 | 11 | 16.20 | 17 | 17.37 | 23 | 14.03 |
| 6 | 33.44 | 12 | 16.20 | 18 | 43.6 | 24 | 31.40 |

**Table 3** Comparison of number of iterations using traditional and current method for Example 5.4.2.

| Loops | Traditional method | | Current method | |
|---|---|---|---|---|
| | Error | No. of iterations | Error | No. of iterations |
| 1 | 0.02 | 9 | 0.006 | 7 |
| 2 | 0.06 | 10 | 0.007 | 8 |
| 3 | 0.03 | 11 | 0.01 | 10 |
| 4 | 0.02 | 12 | 0.006 | 11 |
| 5 | 0.02 | 11 | 0.01 | 8 |
| 6 | 0.01 | 12 | 0.01 | 9 |
| 7 | 0.01 | 13 | 0.01 | 10 |
| 8 | 0.05 | 8 | 0.03 | 6 |

The corrections of the first trial are applied to get a distribution as shown in Fig. 7b.

In a similar way the iterations are done and after third trial the error in first loop is 0.0 and in the second loop is 0.03 which is very small. The corrections are applied and further trials are discontinued by taking this level of distribution to be satisfactory.

The final distribution is, therefore, as shown in Fig. 7c.

This example is given in [4] pp. 449–451 and has been solved in two iterations. The author has randomly guessed the initial trial values of discharges ($Q_2$ = 56, $Q_1$ = 44 & $Q_5$ = 3) without citing any particular reason or method. The guess work for initial discharge is not easy for the beginners; it needs experience to make a guess for initial discharge. But by this approach, beginners can easily solve this problem in three iterations with more accuracy.

**Example 3.** One more general example of pipe networks is shown in Fig. 8. Here the values of three discharges are given and the system is denoted as $TSN_4$. Now the value of its unknown discharges will be $Q_1, Q_2, Q_3, \ldots, Q_{24}$ for pipe 1, 2, 3, …, 24 respectively.

The initial trial value of discharge in each pipe as given in Fig. 8 is calculated from reference Eq. (31) of $TSN_1$. Therefore we get the starting point of iteration in each loop of the pipe network shown in Table 2.

Using the above calculated initial trial point it is found from the Table 3 that after 6–11 iterations the correction $\Delta Q$ is within convergence limits (0.07) in all the loops. And the final values and directions of flow of discharges are obtained.

## 5. Conclusion

Solving reservoirs and pipe networks problem by using topological similarity is simpler because of the following:

1. Concept of topological similarity is successfully applied to a set of two topologically similar pipe networks.
2. Reference system for pipe networks is successfully defined.
3. Solution of reference system is fruitfully used to calculate the value of initial head/ discharge of pipe networks.
4. This procedure eliminates pure guess work for initial head/discharge of pipe networks.
5. Experience for guessing a proper initial head in case of reservoir problems and initial discharge in case of pipe network problems is not required.
6. As solution of reference system can be used to solve all other topological similar systems, it is possible to develop an interactive graphic computer package for calculating initial trial values for head/discharge of pipe networks.
7. The concept of Topological similar system has been successfully brought into the domain of fluid mechanics.

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